Bitcoin, Currencies, and Fragility: Supplementary Discussions

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Rational Expectations

Discretely seen, a price is expected cash flow received at the end of the next period $t+1$ plus expected price at period $t+1$. So let P_t , C_t, and I_t be the price, cash flow (payout to investor) and information, respectively, at period t, with r_d the discount rate. Without any loss, we simplify by assuming C_i and r_d are not stochastic. We note that "cash flow" to investor includes any payout, not just dividend, so C_t includes the liquidation value.

$$P_t = \frac{1}{1+r_d}\left(C_{t+1} + \underbrace{\mathbb{E}(P_{t+1}|_{I_t})}_{\frac{1}{1+r_d}\left(C_{t+2}+\mathbb{E}\left(\underbrace{\mathbb{E}(P_{t+2}|_{I_{t+1}})|_{I_t}}_{\dots}\right)\right)}\right), \tag{1}$$

By the law of iterated expectations,

$$\mathbb{E}\left(\mathbb{E}(P_{t+2}|_{I_{t+1}})|_{I_t}\right) = \mathbb{E}(P_{t+2}|_{I_t}).$$

Allora, noting that, at the present, seen from period t, $\mathbb{E}(P_{t+1}|_{I_t})$ is written as $\mathbb{E}(P_{t+1})$.

$$P_t = \lim_{n\to\infty}\left(\underbrace{\sum_{i=1}^{n}\left(\frac{1}{1+r_d}\right)^i C_{t+i}}_{=0 \text{ for bitcoin}} + \left(\frac{1}{1+r_d}\right)^n \mathbb{E}(P_{t+n})\right), \tag{2}$$

We notice that the second term vanishes under the smallest positive discount rate. In the standard rational bubble model [1] P (actually, its equivalent, the component that doesn't translate into future cash flow) needs to grow around r_d *forever*. Cases of P growing faster than r_d are never considered as the price becomes explosive (intuitively, given that we are dealing with infinities, it would exceed the value of the economy) [2].

As we increase n, additional cash goes into C_{t+n}; in principle, for $n \to \infty$ it must be all cash outside of bubbles.

Earning-free Assets with Absorbing Barrier

Now, bitcoin is all in the second term, with a hitch: there is an absorbing barrier — should there be an interruption of the ledger updating process, some loss of interest in it, a technological replacement, its value is gone forever. As we insist, bitcoin requires distributed attention.

We define the stopping time as $\tau \triangleq \inf\{n>0; P_{t+n}=0\}$, with $P_{>\tau}=0$.

> **Comment 1: Failure rate**
> *Critically the probability of hitting the barrier does not need to come from price dynamics, but from any failure rate — the only assumption here is a failure rate >0.*

So we impose a layer on top of the dynamics.

$$\mathbb{E}(P_{t+n}) = \mathbb{E}(P_{t+n}|_{t+n<\tau})\,\mathbb{P}(t+n<\tau) + \underbrace{\mathbb{E}(P_{t+n}|_{t+n\geq\tau})}_{=0}\,\mathbb{P}(t+n\geq\tau) \tag{3}$$

Let π be the probability of being absorbed over a single period. Rewriting Eq. 1 with no cash flow, i.e. $C_{t+i}=0\ \forall i$, and eliminating cases for which the expectation is infinite:

$$P_t = \frac{1}{1+r_d}\left((1-\pi)\underbrace{\mathbb{E}(P_{t+1}|_{I_t}|_{(t+1)<\tau}))}_{\frac{1}{1+r_d}\left((1-\pi)\ \underbrace{\mathbb{E}(P_{t+2}|_{I_t}|_{(t+2)<\tau})}_{\dots}\right)}\right). \tag{4}$$

We therefore have

$$P_t = \lim_{n\to\infty}\left(\frac{1-\pi}{1+r_d}\right)^n \mathbb{E}(P_{t+n}|_{(t+n)<\tau}) = 0 \tag{5}$$

For the price to be positive now, P_t must grow *forever*, exactly at a gigantic exponential scale, $e^{n(r+\pi)}$, without remission, and with total certainty.

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Comment 2: The problem of P_∞

The argument that P can grow faster than $e^{n(r+\pi)}$ for a while and accumulate valuation is insufficient: once it stops growing, by backward induction, future absorption makes P_t valued at 0. Remember that we are dealing with infinities.

Furthermore variable mortality rates makes the needed growth vastly in excess of both rates r_d and π. Let π be stochastic with realizations $\pi(1+a)$ and $\pi(1-a)$ — two Diracs at the mean deviation of π. Then the required growth rate must be $e^{n(r+\pi+\sigma)}$, where $\sigma = \frac{\log(\cosh(\pi a n))}{n}$, an additional convexity term $\sigma \approx a\pi$.

Some wrinkles concerning Lindy survival

Now what if instead of a fixed π of absorption, we had a "Lindy" π, that is declining in time?

Allora, redoing the exercise, with π_i variable: $\left(\frac{1-\pi}{1+r_d}\right)^n$ becomes $\left(\frac{1}{1+r_d}\right)^n \prod_{i=1}^n (1-\pi_i)$.

For P_t to be positive it would still need to grow until period n at the discount rate plus the geometric mean $\pi' = 1 - \left(\prod_{i=1}^n (1-\pi_i)\right)^{1/n}$, that is, at an exponential *forward* rate $e^{n(r_d+\pi')}$, once again without remission, and with total certainty.

For, again, we note that any growth rate in excess of r_d leads to, in the absence of absorption, bitcoin to displace the rest of the economy.

Note on Ricardo Perez-Marco's ("Ricardo Doorknob") Comments

I would not have singled out Perez-Marco (hence "Ricardo doorknob" owing to the fact that his understanding of financial matters equals those of a doorknob –and that includes financial mathematics), nor exposed his charlatanism, except that he is abetting a smear campaign on yours truly. And he is commenting on the above in bad faith.

The fact that the π are declining in the text above means the blowup probabilities can, as I said, decline, lengthening the life expectation as time goes by. Critically, as Ricardo doorknob doesn't seem to get, they do not need to reach 0. It depends on the limit (constant probability gives an expected survival of $\frac{1}{p}$). For instance we have $\lim_{n\to\infty} 1-\left(\prod_{i=1}^n \left(1-\pi\left(1-\frac{1}{i^\alpha}\right)\right)\right)^{1/n} = \pi$, which, assuming α positive is Lindy down to a local probability π –and the distribution has a constant memoryless life expectancy at infinity but never preasymptotically.

Allora, the conditional expectation of stopping time conditional on having survived to m will increase locally.

$$\mathbb{E}(\tau|_{\tau>m}) = \frac{\sum_{t=m}^{\infty} t\,\pi\left(1-\frac{1}{t^\alpha}\right)\prod_{i=1}^{t-1}\left(1-\left(1-\frac{1}{i^\alpha}\right)\pi\right)}{\sum_{t=m}^{\infty} \pi\left(1-\frac{1}{t^\alpha}\right)\prod_{i=1}^{t-1}\left(1-\left(1-\frac{1}{i^\alpha}\right)\pi\right)}$$

Anyway, poseurs-mathematicians like Ricardo doorknob or not, the probability will never be 0 in real time, and that's critical. Unlike in math, the real world is both preasymptotic and imperfect. Everything has a hazard rate, whether we can identify the cause or not, and unless it finds some real use in a hurry, BTC is worth exactly 0.

References

[1] O. J. Blanchard and M. W. Watson, "Bubbles, rational expectations and financial markets," *NBER working paper*, no. w0945, 1982.

[2] M. K. Brunnermeier, "Bubbles," in *Banking Crises*. Springer, 2016, pp. 28–36.